

Total No. of Questions - 24
Total No. of Printed Pages - 3

Regd.
No.

Part - III
MATHEMATICS, Paper - I (A)
(English Version)

Time : 3 Hours

Max. Marks : 70

Note: This question paper consists of THREE Sections A, B and C.

SECTION - A

10×2=20

I. Very Short Answer Type Questions.

- (i) Answer ALL questions.
(ii) Each question carries TWO marks.

1. If $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that
$$f(x) + f\left(\frac{1}{x}\right) = 0$$

2. Find the domain of the real valued function $f(x) = \frac{1}{(x^2-1)(x+3)}$

3. If $\begin{pmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ -2 & a-4 \end{pmatrix}$, then find the value of x, y, z and a .

4. If ω is complex (non-real) cube root of 1, then show that -

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$$

5. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j}$. Find the unit vector in the direction of $\vec{a} + \vec{b}$.



6. Find the vector equation of the line passing through the point $2\hat{i} + 3\hat{j} + \hat{k}$ and parallel to the vector $4\hat{i} - 2\hat{j} + 3\hat{k}$.

7. If the vector $2\hat{i} + \lambda\hat{j} - \hat{k}$ and $4\hat{i} - 2\hat{j} + 2\hat{k}$ are perpendicular to each other, find λ .

8. Prove that $\sin 78^\circ + \cos 132^\circ = \frac{\sqrt{5}-1}{4}$

9. Find $\sin^2 82\frac{1^\circ}{2} - \sin^2 22\frac{1^\circ}{2}$

10. Show that $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \log_e 3$

SECTION - B

5×4=20

I. Short Answer Type Questions.

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. If $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, then show that $(aI + bE)^3 = a^3I + 3a^2bE$, where I is unit matrix of order 2.

12. $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors. Prove that the following four points are coplanar – $\vec{a} + 4\vec{b} - 3\vec{c}$, $3\vec{a} + 2\vec{b} - 5\vec{c}$, $-3\vec{a} + 8\vec{b} - 5\vec{c}$, $-3\vec{a} + 2\vec{b} + \vec{c}$.

13. Prove that for any three vectors $\vec{a}, \vec{b}, \vec{c}$

$$[\vec{b} + \vec{c} \quad \vec{c} + \vec{a} \quad \vec{a} + \vec{b}] = 2 [\vec{a} \vec{b} \vec{c}].$$

14. Prove that $\tan 70^\circ - \tan 20^\circ = 2 \tan 50^\circ$

15. Solve that equation $\sin x + \sqrt{3} \cos x = \sqrt{2}$

16. Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$

17. In ΔABC , if $a : b : c = 7 : 8 : 9$, then find $\cos A : \cos B : \cos C$



III. Long Answer Type Questions.(i) Answer **ANY FIVE** questions.(ii) Each question carries **SEVEN** marks.

18. If $A = \{1, 2, 3\}$, $B = \{\alpha, \beta, \gamma\}$, $C = \{p, q, r\}$ and $f : A \rightarrow B$, $g : B \rightarrow C$ are defined by $f = \{(1, \alpha), (2, \gamma), (3, \beta)\}$, $g = \{(\alpha, q), (\beta, r), (\gamma, p)\}$, then show that f and g are bijective functions and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

19. Using mathematical induction, show that

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots \text{ upto } n \text{ terms} = \frac{n}{3n+1} \quad \forall n \in \mathbb{N}.$$

20. Show that

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

21. Solve $x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$ by using matrix inversion method.

22. If $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{c} = -\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{d} = \hat{i} + \hat{j} + \hat{k}$, then compute $|(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})|$

23. If A, B, C are the angles of a triangle, prove that

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$$

24. If $r_1 = 2$, $r_2 = 3$, $r_3 = 6$ and $r = 1$, prove that $a = 3$, $b = 4$ and $c = 5$.

